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Quantum mechanical indeterminacy

1. Introduction

One of the basic quantum mechanical features is a novel uncertainty concept. While in classical physics uncertainty is equivalent to lack of knowledge – any physical quantity has a determined value at any instant of time, but usually we don't know it and hence must resort to a statistical description –, the quantum mechanical uncertainty is an intrinsic property of microscopic systems. In fact, it is a new logic category that goes beyond the simple either-or scheme (which is so efficient at mastering our life). A typical example of quantum mechanical indeterminacy is the path uncertainty of a photon (likewise, an electron or a neutron) in an interferometric device, which forbids us to visualize the particle as being localized.

Actually, nonlocality is well known also from classical wave theory, especially from optics. In an interferometer, an incoming wave will be divided, by a beamsplitter, into two coherent parts which travel along spatially separated paths, however, contrary to the quantum mechanical picture, both parts are 'objectively real'. In particular, a measurement on one partial wave will in no ways affect the other.

Generally speaking, in the quantum mechanical formalism uncertainty is a direct consequence of the operator character of the observables. Notably canonically conjugate variables such as position (x) and momentum (p) are represented by Hermitian operators that fulfil canonical commutation relations

$$[x, p] \equiv xp - px = i\hbar 1, \qquad (1)$$

where $\hbar$ is Planck's constant divided by 2π. From this equation it is easily proved (see for instance [5]) that the uncertainties of x and p, Δx and Δp, are bound to satisfy the famous Heisenberg uncertainty relation

$$\Delta x \cdot \Delta p \geq \frac{\hbar}{2}. \qquad (2)$$

It is important to note that this condition applies to pure states, in contrast to classical theory in which $\Delta x = \Delta p = 0$ for a pure state. It becomes obvious from equation (2) that x and p cannot be simultaneously sharp. This is in agreement with the experimental fact that you cannot directly measure both x and p simultaneously on one and the same system. (The same holds true for any pair of conjugate variables.) You have to make a decision whether you want to measure x or p, and this measurement will strongly disturb the system.

So we have to put up with the existence of an intrinsic quantum mechanical indeterminacy. This will not irritate us too much when we think of uncertainties that are of truly microscopic dimensions, as we find them, in particular, in the behavior of electrons bound in atoms or molecules. Here, Heisenberg's uncertainty relation (2) contradicts the classical concept of orbits. But this is a very tiny effect we need not worry about.

However, quantum mechanical uncertainties can extend over manifestly macroscopic dimensions! (Note that the relation (2) is an inequality!) This is the first point that will be treated in the following. Further, the important role uncertainty plays in experiments with entangled systems, will be discussed. Finally, it will be explained how uncertainty might be utilized in quantum computers.

2. Macroscopic position uncertainty

It is well known from classical optics how an interferometer works. An incident light beam is split at the entrance mirror into two beams that travel along different paths and eventually become reunited at the exit mirror. The outgoing light produces an interference pattern whose position is determined by the difference of the phases the two partial beams acquire during their passage through the interferometer. Actually, this phase difference is a geometrical quantity; it is the difference of the armlenghts in units of the wavelength of the light. This is easily understood. However, we run into troubles when the interferometer is operated at extremely low intensities. The intensity may be set, with the help of an absorber, to such a low level that single photons enter the interferometer one after the other, with an average distance that distinctly exceeds the armlengths. So we can safely assume that one photon, at maximum, is present in the interferometer at any instant of time. Now, it is a matter of fact that an interference pattern emerges also in such extreme conditions. (Naturally, one has to choose a very long exposition time.) This means that any individual photon must obey the 'rules': In particular, it must 'know' that there are certain places in the observation screen (corresponding to the intensity minima in the interference pattern) which it should keep clear of, whereas other places (corresponding to the intensity maxima) should be preferably steered for.

As Dirac [1] put it, the photon 'interferes with itself'. However, we cannot understand how it manages to find out the afore-mentioned phase difference when it travels – like a classical particle – along just one arm in any individual case. So the interference phenomenon forces us to assign to any individual photon a position uncertainty in the sense that it is *intrinsically* indeterminate which path it takes in the interferometer. Certainly, this is a macroscopic effect!

The position uncertainty in question is produced in a rather simple way: A beamsplitter, i.e. a semitransparent mirror, does the job. So we need no interferometric device, and hence can make the position uncertainty extremely large. For instance, we can couple the two output channels of a beamsplitter to an optical fibre each. Then the position uncertainty grows linearly with time, due to the propagation of the photon, and this growth is limited in practice only by the absorptivity of the fibre material. This restriction might be removed by letting the photon, after beam-splitting, propagate in the cosmic space. So the position uncertainty might attain really gigantic dimensions.

3. Entangled States

Quantum mechanical uncertainty plays a crucial role in entanglement. This phenomenon is a typical quantum mechanical feature. In fact, any interaction brings the total system in an entangled state. Of special interest are *maximally* entangled states, a typical example being polarization entangled photon pairs (for more details see for instance [4]).

Two photons are emitted simultaneously into different directions so that they can be registered by separate detectors. Their polarization states are entangled, i.e., they are strongly correlated as expressed by the wavefunction

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left(|x\rangle_1\,|x\rangle_2 + |y\rangle_1\,|y\rangle_2\right). \tag{3}$$

Here, $|x\rangle_1$ describes a photon that is linearly polarized in x direction and propagates in direction 1, and so on. As a result of the propagation the two photons become widely separated. But what can be said about their polarizations? The first statement is that the individual polarization is completely indeterminate.

An observer, following a nice tradition let us think of a woman named Alice, that investigates the photons travelling in direction 1 will arrive at the conclusion that what she observes is unpolarized light. For measurement she will utilize a polarizing prism (a birefringent crystal) with a detector placed in each output channel. The prism splits an incident beam into two beams that are linearly polarized in two orthogonal directions. For simplicity, let us assume that the two possible polarization directions are x and y. (Note that Alice is free to rotate her prism at an arbitrary angle, but this will not alter her finding that she is dealing with unpolarized light.) So when the detector in the output channel corresponding to x (y) polarization responds, it indicates the presence of an x (y) polarized photon. (Strictly speaking, the detector 'click' gives us information on the past only, since the photon gets lost in the detection process.) So, what Alice will measure is a sequence of events in which x and y polarizations are indicated with equal probability, at random. This is just a characteristic feature of unpolarized light. A second observer, let us name him Bob, who is analyzing the photons propagating in direction 2, will likewise find the light to be unpolarized.

Things become amazing, however, when both observers, after having completed their measurements, come together and compare their data sets. It might surprise them to see that in any individual case they measured the same polarization (x or y

polarization). This means that the polarization states of the two photons are correlated as strongly as possible.

Let us now assume that one of the observers, say Alice, executes her measurement a little earlier than the other (Bob). Having an a priori knowledge of the entangled state (3), she has got a clairvoyant capability: Having carried out a measurement on 'her' photon, she can predict *with certainty* the outcome of Bob's measurement on 'his' photon, in any individual case. So the indeterminacy of Bob's photon (with respect to its polarization state) has actually disappeared, as a result of Alice's measurement. Quantum theory claims that this happens instantaneously; what takes place is the so-called reduction (or collapse) of the wavefunction induced by measurement. The point is that only one subsystem (photon) is actually subjected to the measurement, and hence affected physically, but nevertheless the quantum state of the total system has drastically changed.

The wavefunction (3) collapses to that part that is compatible with the outcome of the measurement. For instance, when Alice measures x polarization, the wavefunction reduces to

$$|\psi\rangle_{meas} = |x\rangle_1\,|x\rangle_2. \tag{4}$$

So we are confronted with the exciting fact that a distant measurement affects the behaviour of a subsystem in such a way that a variable that is initially uncertain becomes instantaneously perfectly sharp. Actually, this effect lies at the heart of the famous Einstein Podolsky Rosen paradox [2]. One may ask what kind of physics this is. Is it a ghost-like interaction, as Einstein suspected? In fact, it cannot be a physical effect since any physical action cannot propagate faster than light. An important point is that the effect in question cannot be used to transmit signals with superluminal velocity. This would, indeed, violate causality which is one of the pillars the edifice of modern physics rests on. What really happens is a transition from undetermined to determined, and this is no physical process. This is clear in classical physics where indeterminacy is identical with a lack of knowledge. So the mentioned transition is a mental process in which an *existing fact* is taken notice of. Quantum mechanically, one can say that such a transition cannot be followed by observation, simply because an observation on a *single system* can never tell us that the observed variable is uncertain. (Uncertainty can be observed on an ensemble only; it is indicated by the fact that the outcomes of the measurement differ.) This explains also that such a transition cannot be made the basis of superluminal signal transmission.

4. Quantum Computing

Observables have sharp values only when the system is in a corresponding eigenstate. This implies that quantum mechanical uncertainty is described by a superposition of eigenstates. The capability to produce superposition states of a desired form is, in particular, an important prerequisite of quantum computing. It is one of the mechanisms that make quantum computing much more efficient than conventional computing. I would like to explain this in some detail.

Conventional computers use bits, i.e. entities that take on two values ‘1’ and ‘0’, as basic elements that are subjected to logical operations. In a quantum computer one can realize those values by two eigenstates of a suitable observable, for instance, spin or energy eigenstates. Let us focus, for definiteness, on two energy eigenstates, a ground state $|0\rangle$ and an excited state $|1\rangle$ of an ion captured in a linear Paul trap. A manifold of such ions is, in fact, a promising candidate for quantum computers to become realized. An important aspect is that any superposition of the basis states,

$$|\psi\rangle = \alpha\,|0\rangle + \beta\,|1\rangle, \tag{5}$$

can be generated with the help of laser pulses. In equation (5) α and β are complex constants subjected only to the normalization condition $|\alpha|^2 + |\beta|^2 = 1$. Hence the basis system $|0\rangle$, $|1\rangle$ which was named qubit (abbreviation for quantum bit) enables us to store, in the coefficients α and β, a lot of additional information. This storage capability becomes really tremendous when we consider many ions, as they are needed in realistic quantum computers. Be N the number of ions (a typical number might be $N = 100$), then the basis states for the whole system can be characterized by a sequence of binary numbers $x_1, x_2, ..., x_N$. Here, $x_1 = 0$ (1) indicates that the first ion is in the state $|0\rangle$ ($|1\rangle$), and so on. It is advantageous to ‘translate’ this sequence into an integer x by interpreting the numbers $x_1, x_2, ..., x_N$ as the digital representation of x

$$x = x_1\,2^{N-1} + x_2\,2^{N-2} + ... + x_N\,2^0. \tag{6}$$

For given N, the integer x can take on the values 0, 1, 2, ..., $2^N - 1$. So N ions give us the opportunity to realize 2^N states which we will denote by $|x\rangle$. The superposition principle allows us to generate (with the help of tailored laser pulses that are addressed to the individual ions) a superposition state

$$|\psi\rangle = \sum_x \alpha_x\,|x\rangle, \tag{7}$$

where the coefficients, apart from the normalization condition, can be set at will. Thus we can store an incredibly large amount of information in a *single quantum state* of the total system. To do the same in a conventional computer we would, for instance, need the gigantic number of 2^{100} storage places, whereas quantum mechanics does with no more than 100 ions! So the quantum mechanical superposition principle allows for massive parallel processing which is, in fact, one of the reasons for the high efficiency of quantum computers.

However, utilizing quantum theory in this way, we encounter a serious problem: While it is not difficult, at the present state of the art, to encode 2^N complex numbers into the ionic system, we never can read them out! Seemingly, they have only virtual existence. In fact, what we can only do is to measure the (total) energy which gives us just *one* value x, and nothing more. So the numbers α are, in a sense, ‘hidden’ rather than directly accessible. This need not worry us as long we are performing calculations with the state (7) in an *intermediate step*. For instance, a Fourier transformation can be carried out. In this way, use can actually be made of the ‘hidden’ parameters.

What is indispensable, however, is that the final result of the calculation is comprised in just one number x which can actually be read out. For instance, this is so in Grover’s search algorithm [3] where the quantum state of the total system is eventually transformed, through application of a suitable sequence of logic gates, into just that state $|x\rangle$ whose argument x we are seeking. Then the measurement yields x, with certainty, and the problem is solved. It should be noted, however, that the mentioned restriction on the quantum algorithms is severe. Actually, it reduces the mathematical problems that might be successfully attacked with quantum computers to a rather special class, and hence is an unprecedented challenge to mathematicians.

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