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The Cosmic Superstring Scenario

Summary

Our scenario understands the observed hierarchy of astronomical objects as being produced by a cascade of fragmentation of macroscopic, i.e. cosmic superstrings. We rely especially on superstrings with maximum angular momentum J for given mass M (yrast strings). Their ratio $\kappa = J/M^2$ is related to the string tension μ of superstring by $\mu = c/(4\pi\kappa)$. With the choice $\mu \approx 10^{-5}\, c^2/G$ and the assumption of the conservation of the radial distribution in the ensemble of fragments, several results are achieved. (a) The central 10 % of this ensemble is mechanically bound and builds later on astronomical objects of all ranks, which fulfil a mass-angular momentum-relation in concordance with the observed astronomical objects ($\mu \approx 10^{-4}\, c^2/G$). (b) the outer unbound fraction of decay products explains the dark matter fraction, (c) a special regime is suggested when fragments approach $J/M^2 \approx G/c$ which permits the existence of black holes (Kerr limit): we expect a final fragmentation with strong interaction such that again yrast strings are produced. This opens a way to explain the existence and the mass ratio between adjacent ranks in the astronomical hierarchy ($\sim 10^{4.5}$).

Zusammenfassung

Zerfallende kosmische Superstrings erzeugen die astronomische Hierarchie

Wir verstehen die Entstehung der Hierarchie astronomischer Objekte durch eine Kaskade von Fragmentierungen makroskopischer ("kosmischer") Superstrings. Dabei stützen wir uns speziell auf Superstrings mit maximalem Drehimpuls J bei gegebener Masse M (yrast strings). Ihr Verhältnis $\kappa = J/M^2$ steht mit der Stringspannung μ durch $\mu = c/(4\pi\kappa)$ in Beziehung. Mit der Wahl $\mu \approx 10^{-5}\, c^2/G$ und der Annahme der Erhaltung der radialen Verteilung im Ensemble der Fragmente erhalten wir diese Ergebnisse: (a) Die zentralen 10 % des Ensembles sind mechanisch gebunden; sie bilden astronomische Objekte aller Rangstufen, die die beobachtete Masse-Drehimpuls-Relation erfüllen ($\mu \approx 10^{-4}\, c^2/G$). (b) Der nicht gebundene Teil kann die dunkle Materie erklären. (c) Wenn die Fragmente mit ihren J- und M-Werten sich $J/M^2 \approx G/c$ nähern – was die Existenz von schwarzen Löchern erlaubt (Kerr-Grenze) – erwarten wir eine extreme Wechselwirkung bei einem letzten Zerfall, der wieder yrast-Strings produziert. Damit wäre die Existenz der Hierarchie und der Faktor zwischen benachbarten Rangstufen erklärt ($\sim 10^{4.5}$).

Introduction and review of our earlier papers

Since the discovery of the recession of extragalactic nebulae it was natural to think of the universe as having originated from a small region, in the extreme from a kind of particle – be it the *atom primitif* of Lemaître [1] or quantum fluctuations today. It was less usual to imagine the birth of astronomical objects of lower rank – e.g. galaxies – from a small region.

Within all these considerations, the notion of angular momentum was lacking. While this was not an obstacle with respect to the universe, it was forbidding relative to the origin of a galaxy: there was no way to house its angular momentum within a much smaller space and relying on standard physics. This situation has changed with the introduction of superstrings of macroscopic masses and angular momenta.

We have combined the basic physics of such objects with what is known on astronomical objects and especially with observed regularities on their angular momentum.

The development of our ideas started with the recognition of a very general mass M – angular momentum J – relation amongst astronomical objects [2-4], being of the form

(1) $$J \approx \kappa M^2$$

where the observed value of κ is of the order of

(2) $$\kappa_{\rm obs} = 1 M^{-1}_{\odot}\, {\rm AU}^2 / {\rm year} \approx 2 \cdot 10^{-15}\ {\rm g}^{-1}\, {\rm cm}^2\ {\rm s}^{-1} \approx 10^3\ G/c$$

(here $M_{\odot}$ = solar mass = $2 \cdot 10^{33}\,g$, AU = astronomical unit = $1.5 \cdot 10^{13}$ cm, year = $3 \cdot 10^7\ s$; G = gravitational constant, c = velocity of light).

This relation has a larger scatter for some classes, but is obeyed for a large range of about 12 powers of ten in M. It cannot be derived from obvious interrelations for gravitating bodies in equilibrium [5]. This relation has been discussed by Trimble [6] who, while first questioning its significance, concludes that there is something to be explained.

The remarkability of this relation is enhanced by the existence of two fundamental parallel relations:

(a) The maximum value of J/M^2 of black holes is likewise a constant, namely [7, 8]

(3) $$\kappa_{BH} = G / c$$

that is, about 10^3 smaller than (2).

(b) The Regge trajectories of hadrons have a constant slope $\Delta J / \Delta M^2$ of the order $\hbar / ({\rm GeV})^2 = 3.3 \cdot 10^{20}\ {\rm cm}^2\ {\rm g}^{-1}\ {\rm s}^{-1}$ which is 10^{35} higher than κ [9].

When it turned out that superstrings are an interesting possibility to explain the variety of elementary particles, it was especially interesting to note that (c) superstrings in the state of maximum angular momentum J for given mass M also obey a relation of the type above, namely

$$J(M) = \frac{c}{4\pi\,\mu} M^2 + J(o) \tag{4}$$

where μ is the string tension, and the specific numerical factor holds for a closed string; *J(o)* is negligible for our applications. It is convenient to use the term yrast state, used in nuclear physics [10, 11], for the state of maximum angular momentum for a given mass. The approximately coinciding mass-angular momentum relations for astronomical objects and yrast superstrings led to the idea that astronomical objects were built from the decay products of macroscopic superstrings in the early universe [12, 13].

The observations show a sequence of hierarchical levels of objects, separated by a factor N in mass and number with $N \approx 10^{4.5}$. We adopted this scheme in the elaboration of the basic idea into a scenario [14].

Crucial for our arguments are the energies of the ensemble of fragments of a decaying yrast string just after fragmentation. The kinetic energy, being identical with the rotational energy of the string, is

$$T_{rot} = (1 - \frac{2}{\pi})\, M\, c^2 \tag{5}$$

and the gravitational potential energy of the ensemble amounts to

$$\Omega = -\frac{GM^2}{L}\, ln\, N \tag{6}$$

where M is the total mass of the yrast and therefore straight string of extension L, assumed to consist of N equal parts ((3.18) and (4.4) in [14]).

We want to draw attention to the dependence of the gravitational potential energy Ω on the dimension of the object. It is well known that this quantity diverges for a point mass. For surfaces and bodies of higher dimensions ("branes" of all kinds) the corresponding integral converges. In other words, it exists for an arbitrary fine sub-division of the bodies. Only in the case of *one* dimension do we encounter the situation that Ω exists for only a *finite* subdivision of the string. In the later version of our scenario we have used elementary particles as the natural lower end of subdivision. Hence just for *one* dimension we have an explicit connection of the macroscopic and microscopic world. In string theory even macroscopic superstrings are "elementary" but can decay. The smallest pieces of superstring, such as possibly quarks, cannot

decay because there is nothing to decay to. Their stability is essentially quantum mechanical as classically there would be no smallest piece of string.
Within this scheme, the adherence to the approximately coinciding mass-angular momentum-relations ensured the correct sizes of the astronomical objects formed. It remained to be shown that the required time scales are reasonable. The first version of our scenario required a strong energy loss of the proto-objects, which took place by gravitational radiation. The lowest hierarchical step seems to be determined by the fact that solid-body-densities were reached there. The essentials of classical mechanics used in the following are briefly described in the annex.

Changes and new aspects

Since it is observationally firmly established that galaxies and clusters of galaxies contain about ten times more mass than those revealed as luminous matter, we devised a scheme [15] with the assumption that within the fragmentation of a macroscopic superstring, 90 % of the mass belongs to "unbound" pieces leaving the forming object while only 10 % are gravitationally bound, remain together and evolve to the specific object (the unbound pieces are thought to go to the next lower rank).
Whereas the scheme described was purely ad hoc, a further addition introduced a physical argument: we assumed that the fragmentation was not completely chaotic but that the motion of each fragment of string immediately after fragmentation is identical to the motion before fragmentation so that the fragments initially keep their radial arrangement and their contribution to the angular momentum [16].
Clearly then the whole ensemble of fragments has too much angular momentum to form a black hole, but as the angular momentum increases with distance from the centre, a certain inner fraction of fragments will have $J/M^2 = G/c$, the Kerr limit for a black hole.
G/c is about a factor 10^3 below the value for bound objects and such a factor in mass is indeed observed for the black holes of galaxies [17, 18]. While this is only a necessary condition, observations show that the possibility is realized. Moreover, detailed observations of a few supermassive black holes in galaxies show that they rotate not far from the Kerr limit [19, 20]. We consider the following further support of our idea: in our case the rotating masses are initially together. In alternative pictures of mass collecting to form a black hole, one has to take care that these additions take place in a way which produces such a high angular momentum. Random collection at a small target does not.
Finally, we have in some way combined and expanded the two earlier papers. We extended the assumption on the partial preservation of the radial mass and angular momentum of the fragments – called "orderly fragmentation" – to the outer parts of the distribution of fragments [21]. We determined the border between the inner bound part and the outer = unbound part by asking that the mechanical energy of the bound part be zero. By demanding that the masses of the two parts should have the ratio 1:10, we arrived at a true constant $\kappa \approx 10\ \kappa_{obs} \approx 10^4\ G/c$.

Energies
In each case, the kinetic energy T_{rot} of a rotating yrast superstring is about one third of its total energy Mc^2 (see equ. (3.18) in [14]), that is, comparable with the latter. The potential energy Ω depends on the number of constituting pieces to be considered. In our first paper, we considered the value of Ω corresponding to $N \approx 10^{4.5}$ pieces, which led to to $\Omega_E \approx -\frac{1}{150} M_0 c^2$ for the decayed ensemble. We proposed then that the decaying string loses that much energy that $T + \Omega \approx 0$. Since $|\Omega_E| << T_{rot}$, this would mean that a large part of the original energy has to be dissipated. For galaxies and ranks above, we could alternatively follow the decay down to the last units, elementary particles, and estimate Ω_E for the corresponding large numbers N of (final) fragments [21]. This is justified since the time scales of the lower rank constituents of a given object are negligible compared with its own time scale. Hence within the evolution of such an object the decay of its parts has reached the lower end of the hierarchy. Thus we can, for these object classes, use the Ω_E belonging to it and fulfill $T + \Omega \approx 0$ for the bound part of the string.
We determined the final extension of the bound part by the condition that virial equilibrium $2T + \Omega = 0$ should take place. In other words, $E = -T < 0$, which reflects the well-known fact that the total mechanical energy of bound objects is negative. Hence, our approximate $E \approx 0$ for the primordial state does not persist to the state of an astronomical object. However, the energies at this final state are much smaller than the ones at the string state (compare the squares of the pertaining velocities $(200\ \text{km/s})^2/c^2$). Therefore the diminution in E can be interpreted much more easily.
Angular momenta do not change within our scenario. That does not mean that we deny the possibility of secondary changes, e.g. by tidal forces. On large scales, these angular momenta are properties of ensembles of fragments: the subset becoming the bound astronomical object has about 10^{-1} of the J/M^2 of the parent object; the smaller subset able to form a black hole has about 10^{-4} of the whole or 10^{-3} of the bound object (see equ.s (8)-(11) in [21]).

Fragmentation and yrast-state
We do not need to know whether the fragmentation takes place directly into the smallest pieces (of the rank under consideration) or if it occurs via intermediate steps or string pieces.
At a certain step, the number of such fragment ensembles be N_0. They have a mass $M_0 = M/N_0$ and their non-yrast angular momentum is $J_0 = J/N^3$, hence $J_0/M_0^2 = (J/M^2)/N_0 = \kappa/N_0$. Substituting κ by our $\mu = 10^{-5}\ c^2/G$, we can ask at which stage of the fragmentation the critical value $J/M^2 = G/c$ is reached. This is the largest value which enables the formation of a black hole. It is not a sufficient condition and it seems that only a small part of the mass enters into that state. Moreover, the ensembles of fragments have almost the same mass:length-ratio as their parents, therefore their extension is ~ 10^4 above their Schwarzschild radius. But due to the imbalance of their kinetic and potential energy, they will shrink by such a factor and reach the region of strong space curvature. The characteristic time scale is in this case the

free fall time of the initial state. In order to obtain specific front factors, we assume that the equilibrium objects which are built from the fragment ensembles are rigidly rotating flat disks with a spheroidal density distribution (as in [22] and [14]).
Then for the radius r_E of this disk the following relation holds

$$\frac{r_E}{M_0} = \frac{25}{48\pi^3}\frac{1}{\mu^2}\cdot\frac{c^2}{G}\frac{1}{N_0^2} \tag{7}$$

With our $\mu = 10^{-5}\, c^2/G$ we obtain $\frac{r_E}{M_0} = 2G/c^2$ (i.e. the Schwarzschild radius for given M_0) for $N_0 = 0.9 \cdot 10^4$.
The near coincidence of this N_0 with our hierarchy factor $10^{4.5}$ leads us to the conjecture that it is just the neighbourhood to the black hole-situation which defines the hierarchy factor and thereby the whole ladder of ranks. With an additional factor $n \approx 3$ we would arrive at $N \approx n \cdot N_0$. However, it is quite reasonable to expect that already before reaching the black hole values strong relativistic effects operate; at 3 r_E there exists the last circular orbit of light around the black hole and at 6 r_E the last stable circular orbits for massive particles [23]. Asking for the N_0 which delivers 6 r_E we obtain $0.4 \cdot 10^4$ and therefore an additional $n \approx 8$ to represent N. An n of this order is also required to understand the production of yrast strings, as we shall see soon. We recall the special nature of this situation:

(a) The radius r_E of the equilibrium object is the sixfold Schwarzschild radius of a mass M_0.
(b) The angular momentum (and thereby rotation period and velocity) are not far from the one of an extremely fast rotation black hole (the Kerr limit).

Amongst the strong interactions at this state, one can think of resonances between rotation periods and orbital periods of neighbouring fragments: the torques between rotating ‘bars’ are much more variable than those between spheroids considered in the case of the Lense-Thirring-effect. With our value of μ this happens at a fraction $1/N_o = 10^{-4.2}$ of the original string or $10^{-3.2}$ of the bound part. Up to this point classical mechanics can be used. Especially, the proper angular momenta of N_o pieces vary with $(N_o^{-1})^3$ of the parent string and their J/M^2 with N_o^{-1} because their rotational velocity is N_o^{-1} times smaller than c. We had assumed that they regain the yrast-state of their parents. It does at least not violate conservation laws, if we assume more specifically that the transfer takes place with constant energy = (rest mass) $\cdot\, c^2$ + rotational energy. As noted earlier (equ. (3.18) in [14]), a rotating yrast-string houses about two thirds of its total mass in form of rest mass and one third as kinetic (rotational) energy. A slow rotating fragment (with $v \approx N_o^{-1}\, c \approx 100$ km/s) would contain a negligible amount of mass in the form of kinetic energy. If this fragment is able to convert 1/3 of its mass into kinetic energy, it could rotate with c and would then be in the yrast-state. If the interactions are so intense that the rest mass acts as a degree of freedom, one could consider this as a kind of equipartition of energy.

We assume that at this stage the interaction between nearby fragments is so strong that they not only break up into a few (say $n \approx 8$) last fragments but also that these pieces reach again the yrast state. As we have seen, this is not an energetic problem, if the kinetic energy can be drawn from the rest mass. There remains the question of the angular momentum. The vectorial sum of the angular momenta of these last step fragments should obey and can obey the value given for the fragments before the last break-up. That means that the sum should be nearly zero (compared with yrast values). If the sum is to be zero and the directions of the J vectors should not be constrained otherwise, at least 4 fragments are required. This is conform with our assumption $n \approx 8$. Because the off-spring of such a last interaction cannot be, in general, identified amongst the present astronomical objects, it seems impossible to check the almost vanishing sum of the angular momenta vectors. As a weaker consequence, however, we predict that the angular momenta of genetically related objects should in general have quite deviating directions. This refers to "brothers", "parents", "children" etc. The member galaxies of the Local Group, e.g., are thought to be such a sample. Their members do not reveal any preferential direction of their poles [24] . The two dominant members are our Galaxy and theAndromeda Nebula. Their angular momenta are not in any special orientation with respect to each other. Moreover, a plane of satellites around the Andromeda nebula and the plane of the Milky Way are about perpendicular; and so is the plane of satellites of the Milky Way with regard to the latter [25]. On the stellar level, the orbits of exoplanets do not show a clear preference for their central star's equator [26]. Especially, the orbits of hot Jupiters are not firmly related to the equators of their central stars [27]. The orbits with known orientation of visual binaries do not show a preference for their poles [28]. Our own ecliptic is strongly inclined to the Milky Way.

Together, N_o and n provide the hierarchy factor $N = N_o \cdot n = 10^{4.5}$.

Before, N was an empirically defined but otherwise ad hoc factor. With the considerations above, a certain tentative explanation is given.

We had ensured that the new gravitational time scales are still reasonable. As the time for the transit from the narrow string state to the expanded state of the astronomical object, they are "free throw" times, identical with free fall times.

Relation with the recently detected gravitational waves

The recently detected cases [29, 30] fit very well with predictions for the merger of two black holes. In our scenario, string ensembles able to form black holes may, in their last stages, have turned into two black holes. The reported angular momentum of the merged black hole is not far from the Kerr limit (0.7 of it) and points to a typical origin from the side of high angular momentum progenitors. Regarding its mass of ~ 60 solar masses, we would attribute this case in our scenario to the rank of star clusters where black holes with masses $\leq$/1000 of the cluster mass could form, according to the experience with galaxies probably in the centres. The typical star cluster mass in our scenario amounts to $10^{4.5}$ solar masses and 10^{-3} of it to 30 solar masses (all at these figures with at best one power of 10 uncertainty because of the schematic figures). Since the observations stem from a region with ~ 10^5 Milky Way galaxies, which amount to ~ $6 \cdot 10^{11}$ stellar clusters of the mass above, we expect for a constant formation rate over an age of $2 \cdot 10^{11}$ years about 3 cases per year within the observed region. This should only prove that our scenario is not in obvious contradiction with regard to the frequency of this process.

The neighbouring ranks in our scenario could also produce black holes and thereby – or in nearby situations – gravitational waves. However, the next lower rank would have $10^{4.5}$ times faster frequencies (and lower amplitudes) making the detection today impossible. The next higher rank events (corresponding to the black holes in the cores of galaxies) would have correspondingly stronger amplitudes but are rare and have frequencies which require a very long term stability of detectors. Therefore at present they cannot be found.

Recalibration of quantities; new tables

In this chapter, we will provide the quantitative consequences of the changes described above, first in principle and then as new tables for the various ranks of the hierarchical scheme. They replace the ones in [14].

The root of the changes consists in the increase of the value of κ by a factor of ten. The latter means a concomitant decrease in the string tension μ by a factor of ten (see [14] equ. (3.5)). Specifically the value of μ was estimated to be be $\mu \approx 10^{-5}\ c^2 / G = 1.2 \cdot 10^{39}$ Newton.

Hierarchy index	Mass M	Extension L		Period of rotation	
	[lg($M/M_\odot$)]	[lg(L/cm)]	[lin. value]	[lg(τ_{rot}/s)]	[lin. value]
7	19	28.8	$2 \cdot 10^4$ Mpc	18.8	$2 \cdot 10^{11}$ years
6	14.5	24.3	600 kpc	14.3	$6 \cdot 10^6$ years
5	10	19.8	20 pc	9.8	200 years
4	5.5	15.3	130 AU	5.3	2.3 years
3	1	10.8	0.8 $R_\odot$	0.8	6 s
2	-3.5	6.3	20 km	-3.7	0.5 ms
1	-8	1.8	60 cm	-8.2	6 ns
0	-12.5	-2.7	20 μm	-12.7	0.5 ps

Table 1: Parameters of whole strings

We leave the masses of our observable objects unchanged (table 1). The masses of the whole strings from which they originated – as inner parts – were 10 x larger. The extensions of these whole strings were then 10 x larger because of the masses and another factor 10 because of the decrease in μ (see equ. (3.6) in [14]), together 10^2 larger. The extensions of the bound parts were 10 x larger than for the same masses before ([14], table 2). The period of rotation is $\tau_{rot} = (2\pi/3)\,(L/c)$ – see (3.14) in [14] – and hence increases with L by 10^2.

Hierarchy index	Mass M		Extension L		Free fall (throw) times τ_{ff}
	[lg($M/M_\odot$)]	M	[lg(L/cm)]	L	[lg(τ_{ff}/year)]
7	18	10^{18} M⊙	27.8	$2 \cdot 10^3$ Mpc	15.3
6	13.5	$3 \cdot 10^{13}$ M⊙	23.3	60 kpc	10.8
5	9	10^9 M⊙	18.8	2 pc	6.3
4	4.5	$3 \cdot 10^4$ M⊙	14.3	13 AU	1.8
3	0	1 M⊙	9.8	0.08 R⊙	-2.7
2	-4.5	10 M_E	5.3	2 km	-7.2
1	-9	$3 \cdot 10^{-4}$ M_E	0.8	6 cm	-11.7
0	-13.5	$6 \cdot 10^{16}$ kg	-3.7	2 μm	-16.2

Table 2: Parameters of the bound parts
(M (Earth) = $M_E = 5.973 \cdot 10^{24}$ kg; $M_\odot = 1.989 \cdot 10^{30}$ kg $= 3.33 \cdot 10^5\ M_E$)

Now we come to the astronomical objects (table 3) and to the transition from the inner fraction of the string towards them. The bound fraction of the string was defined by the border value of the mechanical energy $E_b = T_b + \Omega_b \approx 0$, while the present astronomical objects (index a) are in gravitational equilibrium and fulfill the virial theorem $2T_a + \Omega_a = E_a + T_a = 0$, hence $E_a < 0$ because $T_a > 0$.

Hierarchy index	Name	Characteristic dimension [lg (R/cm)]	'Spherical' density [lg(ρ/g cm^{-3})]
7	–	29.9	-39.0
6	Superclusters, rich clusters of galaxies	25.4	-30.0
5	Galaxies	20.9	-21.0
4	Stellar clusters	16.4	-12.0
3	Double stars and stars with planetary systems	11.9	-3.0
2	Planet-satellite-systems	7.4	(+6.0)
1	Moons. asteroids. comets	2.9	(+15.0)
0	–	(-1.6)	(+24.0)

Table 3: Astronomical objects

It is clear then that from the energy E_b of the inner string part until the final E_a of the astronomical objects, some energy loss must have taken place. However, this energy loss is small compared with the string values as we noted above. Therefore, it is an approximation comparable with our other ones to say that in this way the bound string part needs no (considerable) energy dissipation to form astronomical bodies.
The characteristic time scales for the evolution from the bound string part to the extended state of present objects are "free throw" times. Although the bound parts of the strings are now 10 x larger, this is still very small compared with the extensions of present objects. Since those are kept constant, the free throw times, equal to free fall times, likewise have the old values but are repeated here for completeness.
Similarly the characteristic dimensions of the astronomical (proto-) objects as estimated from their masses and angular momenta by equ. (6.1) from [14], are the same

as before and so are the densities for an assumedly spherical form. Hence our earlier remarks suggesting that the lower end of the hierarchy ladder is due to impossible densities do still apply.
With regard to the upper end of the hierarchy steps in our tables (rank = 7) more will be said in a forthcoming paper. Equation (4) holds for strings in Minkowski space. De Vega and Sanchez [31] have shown that in de Sitter space equation (4) holds for small M but for large M there are important corrections so that J reaches a maximum and then rapidly decreases and thus the observable universe would not be in conflict with the limits on the rotation of the universe [32]. Another possibility would be an at least creation as pairs with zero net angular momentum in concordance with our view what happens near to $J/M^2 \approx G/c$. Especially so, if the 'upper end' is only a station in an infinite sequence from above.

Appendix

<u>*Planck quantities*</u>
It is well known that one can derive from Planck's constant $\hbar = 1.05 \cdot 10^{-27}\, g\, cm^2\, s^{-1}$ with the aid of other constants (G = constant of gravity, c = velocity of light, k = Boltzmann's constant) the Planck mass

$$m_P = \sqrt{\hbar c} \,/\, G = 2{\cdot}10^{-5}\, g = \quad (=1.2 \cdot 10^{19}\, GeV)$$

the Planck length

$$l_P = \sqrt{\hbar G \,/\, c^3} = 2 \cdot 10^{-13}\ cm$$

the Planck time

$$t_P = \sqrt{\hbar G \,/\, c^5} = 5 \cdot 10^{-44}\ s$$

the Planck density

$$\rho_P = c^5 \,/\, (\hbar G^2) = 5{\cdot}10^{93}\, g\, cm^{-3}$$

and the Planck temperature

$$T_P = \sqrt{\hbar c^5 \,/\, G \,/\, k} = 1.4 \cdot 10^{32}\ \text{Kelvin.}$$

Since the ratio J/M^2 plays such a crucial role in our scenario, it seems of interest to ask for its Planck value. Remarkably, its value turns out to be the limiting value for black holes

$$(J \,/\, M^2)_P = \hbar \,/\, m_P^2 = G \,/\, c = \kappa_{BH}$$

and hence independent of ħ!

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Annex

Principles of the state and evolution of self-gravitating systems

Systems of self-gravitating bodies which interact only gravitationally with themselves keep their mechanical energy E and their angular momentum J. Their mechanical energy E consists of the kinetic energy T and the – negative – potential energy Ω:

(A1) $$E = T + \Omega \qquad T = \frac{1}{2}\sum_i m_i\, v_i^2 \qquad \Omega = -G\sum_{i<j} \frac{m_i\, m_j}{r_{ij}}$$

Herein, the m_i are the masses (approximated as points) v_i their velocities and r_{ij} the distance between m_i and m_j.

A system with $E < 0$ remains within certain bounds, in short, it is bound. Otherwise, it can expand to infinity. While E = const for an isolated system, the distribution of E into T and Ω is a priori free. However, for bound systems, the time averages of T and Ω observe the virial theorem

(A2) $$< 2T > + < \Omega > = 0. \qquad (< > \text{ means the time average})$$

Systems with strong enough interaction tend towards an equilibrium state (they are "virialized") in which the deviations of T and Ω from their time averages are small. Obvious consequences from (A2) are

(A3) $$< T > = < -\Omega > \text{ and } E = < \Omega >$$

If the virial theorem is essentially violated, its establishment needs a certain time. The experience from many n-body calculations shows that after only a few free fall times this overall kind of equilibrium is reached (the 'finer' equilibria of the velocity-space-distribution need much more time). The free fall time τ_{ff} can be estimated with the one of a homogeneous sphere of mass m and radius r:

(A4) $$\tau_{ff} = \frac{\pi}{2\sqrt{2}}\sqrt{\frac{r^3}{\mathrm{G}m}}$$

Since the longest time scale is always the governing one, the free fall time of the given configuration applies in case of shrinking, while the one of the end state is appropriate in case of expansion (free throw time).

The angular momentum is a vector $J = \sum_i m_i\, r_i \times v_i$ but we shall deal only with its absolute amount $|J|$ and use J for it. If not said otherwise, the angular momentum around the barycentre of the body is understood.

For theoretical purposes it is easier to treat examples with spherical symmetry; this can lead to singularities. Moreover, in cosmic situations, a zero angular momentum is almost impossible. Except for pathological situations (a small part of an ensemble on the very outside could carry much angular momentum), a certain amount of angular momentum corresponds to a certain size of a self-gravitating system in equilibrium. The exact value depends on the mass distribution within the system but the order of magnitude is defined. E.g., we can project the mass distribution of a homogeneous sphere with mass *m* and radius *r* into the plane and discuss the pertinent quantities for such a disk. For the kinetic energy, rigid rotation of the disk is assumed. Thereby one can express the kinetic energy and the angular momentum with the maximum (edge) velocity of rotation v_{rot} and then the first with the latter.
In this case we have

$$T = \frac{5}{4}\,\frac{J^2}{mr^2} \quad \text{and} \quad \Omega = -\frac{3\pi}{10}\,G\,\frac{m^2}{r} \tag{A5}$$

Consequently

$$r = \frac{1280}{27\pi^3}\,\frac{J^2}{Gm^3} \approx 1.529\,\frac{J^2}{Gm^3} = 1.529\left(\frac{J}{m^2}\right)^2 m/G \tag{A6}$$

The present configuration is the one with the deepest total energy amongst the sequence of rigidly rotating spheroids.
Thus this *r* is the approximate minimum radius of a self-gravitating system in equilibrium which can house the given angular momentum. It depends only on *J* and *m* and characterizes an end of a system's evolution when energy losses lead to the deepest state possible with the given *J*.